Gyro-odd Elastodynamics

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Metamaterials containing active constituents from gyroscopic and odd-elastic effects challenge some basic symmetry principles of classical elasticity, which has significantly broadened the foundation of possibilities to manipulate non-reciprocal and non-Hermitian elastic waves in a highly unconventional guise.

The past decades have witnessed an extensive interest in the exploration of artificial metamaterials like Cosserat elasticity, Willis metamaterials, pentamode metafluid and many others that go beyond the framework of conventional elastic solids. Beyond the passive media, lately the topic of active metamaterials/lattices relying on external driven systems is getting hot. On one hand, active ingredients give new perspectives to reexamine the basic assumptions and principles, for instance, energy conservation, time-reversal symmetry, reciprocity, etc., that were disregarded in the past; on the other hand, they open new avenues to material parameters modulation and energy exchange, giving rise to rich non-reciprocal and non-Hermitian wave phenomena and applications.

With regard to active metamaterials, gyroscopic media and odd elasticity are two fascinating examples that have aroused broad interest in topological physics due to the breaking of time-reversal symmetry and Maxwell-Betti reciprocity, respectively. The two active effects have been harnessed separately to manipulate one-way wave propagation and non-Hermitian mode confinement. Yet it remains to be explored, beyond the purely gyroscopic and odd-elastic effects, what implications gyro-odd coupling (see Fig. 1) has for wave propagation in general elastodynamic systems with inertial terms.

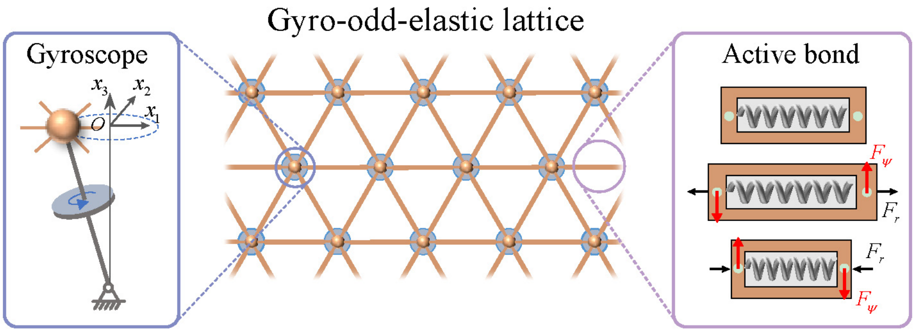

Figure 1 Active lattice comprising gyroscopic and odd-elastic effects. The lumped masses are mounted on spinning gyroscopes to harness gyroscopic coupling. Active bonds that can generate non-conservative forces fuel the odd-elastic response.

In our recently published paper in Communications Materials, a systematic study has been conducted in the framework of two-dimensional gyro-odd continuum mechanics. Specifically, the gyroscopic coupling is effectively mapped into a density tensor of imaginary off-diagonal components, while the non-conservative odd interaction generalizes the Hooke's law such that the stiffness matrix now acquires asymmetric components, i.e., odd-elastic moduli A and K^{o} for isotropic solids. In our paper, we explored the non-Hermitian wave dynamics and stability transitions resulting from odd-elastic energy exchange in unbound media. Beyond that, we also unveil highly resistant non-Hermitian Rayleigh surface waves in semi-infinite space which, phenomenologically speaking, display features reminiscent to chiral edge states in topological Chern insulators.

Our work reveals how the gyro-odd media extract energy from odd-elastic engine cycles comprising remarkable features of stability transitions. The gyroscopic effect alone cannot contribute to non-Hermitian features because it has no influence on the energy dissipation. A purely odd elasticity can experience spectral singularity when the non-conservation active force starts to generate work through the odd-elastic engine cycle. However, for the combined gyro-odd materials, such engine cycle always sustains due to the presence of gyroscopic effect which can drive a piece of material moving along an elliptical orbit (see Fig. 2), and finally give rise to a gain or a loss of elastic energy.

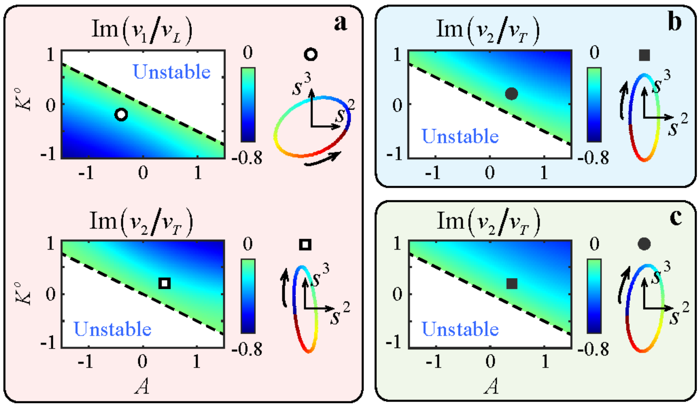

Figure 2 Gyro-odd elastodynamics in unbound media. **a** Phase diagrams showing the stability transition in the odd-elastic parameters space when the level of gyricity $\alpha = 0.5$. The colored (stable) regions depict the imaginary parts of phase velocities $v_{1,2}$. The dashed lines mark the transition boundary, that is $A + 2K^{o} = 0$. As indicated by the symbols, some odd-elastic engine cycles are provided in the strain subspace (s^2, s^3) to analyze the mechanical work done on the solid. Likewise, (**b**, **c**) present the results for the cases $\alpha = 1$ and $\alpha = 2$, respectively, but now only the v_2 branch exists.

Perhaps most importantly, we showcase non-Hermitian Rayleigh waves that by the right choice of odd-elastic moduli travel along a strict unidirectional path, where the travel distance and degree of confinement, likewise can be tuned. Thanks to the resulting absence of reciprocity, such kind of highly controllable and non-reciprocal Rayleigh waves that can traverse through varied bends and obstacles with suppressed back-scattering and reduced interference from bulk waves (see Fig. 3). These unusual behaviors are unreachable with either gyroscopic or odd-elastic effect separately.

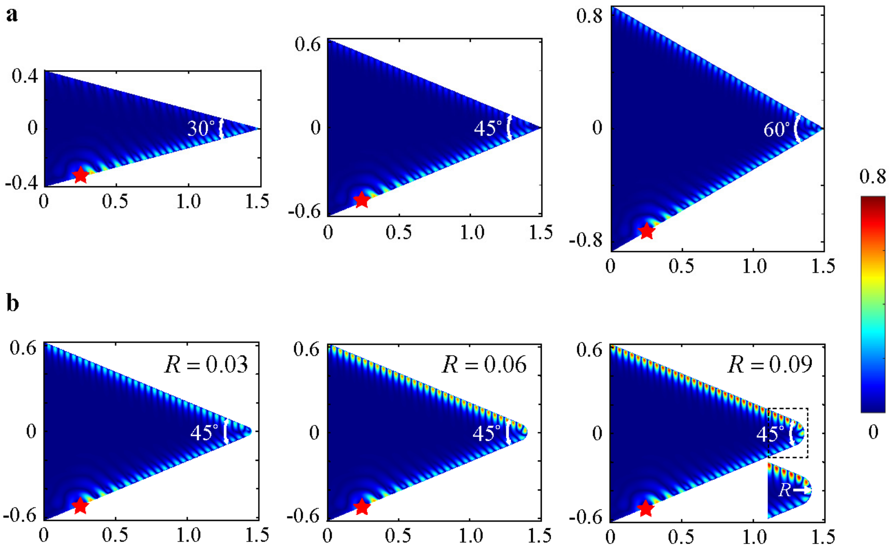

Figure 3 Rayleigh waves in wedges of different vertex angles and curvature radii. **a** Simulated displacement fields when the wedges are excited by local vibrations (red stars). The fields are simulated by applying absorption boundary conditions at the left terminations. Likewise, **b** shows the results for wedges whose vertex curvature radii take values R = 0.03, 0.06 and 0.09.

The synergistic interplay between the gyroscopic and odd-elastic response provides remarkable degree of tunability to manipulate the non-Hermitian and non-reciprocal elastic waves. We envision that the gyro-odd materials and the unusual physics can stimulate further explorations in active devices using, for instance, robotics and piezoelectric metamaterials, aiming at robust and scattering-immune guiding of surface acoustic waves.

For more information, please refer to our paper published in Communications Materials: https://doi.org/10.1038/s43246-022-00297-5
Gao, P., Qu, Y. & Christensen, J. Non-Hermitian elastodynamics in gyro-odd continuum media. Commun. Mater. 3, 74 (2022).